Application of Zeller's Congruence and Combinatorics for Annual Leave Optimization

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***Abstract*—Annual leave planning is an important issue for employees who wish to maximize their vacation periods while minimizing the number of leave days used. This paper proposes an approach for optimizing employe's annual leave by applying Zeller's Congruence and combinatorial analysis. Zeller's Congruence is utilized to determine the day of the week for any given date. In addition, combinatorial methods are employed to evaluate possible combinations of leave days around weekends and public holidays. The proposed method identifies leave schedules that maximize consecutive vacation days under a limited leave quota. To demonstrate the effectiveness of the approach, a case study based on Indonesia public holiday calendar is conducted. The results indicate that mathematical techniques can effectively support annual leave planning and schedule optimization.**

Keywords*—*annual leave optimization, Zeller's Congruence, combinatorics, calendar computation, discrete mathematics.

I. INTRODUCTION (*HEADING 1*)

A. Background

Annual leave is a form of employee entitlement that allows workers to take paid time off while maintaining their employment status and salary [1], [2]. According to Indonesian labor regulations, employees who have worked continuously for at least twelve months are entitled to annual leave benefits [2]. Besides fulfilling legal requirements, annual leave contributes to employee well-being, work-life balance, and productivity.

Many employees aim to maximize their vacation periods by strategically placing annual leave around weekends and public holidays. However, determining the most effective leave schedule is not always straightforward, especially when the available leave quota is limited. Employees often select leave dates based on intuition rather than systematic analysis, which may result in inefficient leave utilization.

This problem can be viewed as a discrete optimization problem. Given a limited number of leave days and a calendar containing weekends and public holidays, the objective is to determine which working days should be taken as leave in order to maximize the total number of consecutive vacation days.

Mathematical techniques provide a systematic solution to this problem. Zeller's Congruence can be used to determine the day of the week corresponding to any calendar date [3], [4]. Meanwhile, combinatorial methods enable the generation and evaluation of possible leave-day selections [5], [6]. By combining these approaches, an employee can identify leave schedules that maximize vacation periods while minimizing leave consumption.

B. Problem Statement

This study investigates the following research questions:

1. How can Zeller's Congruence be applied to determine weekdays in a calendar year?
2. How can combinatorial methods generate feasible leave schedules?
3. How can these methods be integrated to optimize annual leave allocation?

C. Research Objectives

The objectives of this research are as follows:

1. To implement Zeller's Congruence for determining the weekday corresponding to a given date.
2. To develop a combinatorial model for generating leave schedules.
3. To identify leave allocations that maximize vacation periods under a limited leave quota.?

II. THEORETICAL FRAMEWORK

A. Calendar Systems and Day Computation

Calendar computation is an important topic in mathematics and computer science because many scheduling and planning problems depend on accurate date calculations. One common problem is determining the day of the week associated with a given date.

In this research, the Gregorian calendar is used as the basis for all calculations because it is the internationally accepted civil calendar system. Accurate identification of weekdays is necessary to distinguish between working days, weekends, and public holidays.

B. *Zeller's Congruence*

Zeller's Congruence is a mathematical algorithm developed by Christian Zeller for determining the day of the week corresponding to a specific Gregorian calendar date [3], [4]. The method is computationally efficient and can be applied to any date without requiring a complete calendar table.

For Gregorian dates, Zeller's Congruence is expressed as [3], [4]

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor + 5J\right) mod 7 \quad (1)$$

where:

h = day of the week,

q = day of the month,

m = month,

K = year within the century,

J = zero-based century.

The resulting value of h corresponds to a specific weekday, where 0 represents Saturday, 1 represents Sunday, 2 represents Monday, and so forth [3].

In this study, Zeller's Congruence is used to classify dates into working days and non-working days, forming the basis for leave optimization.

C. *Combinatorics*

Combinatorics is a branch of discrete mathematics concerned with counting, arrangement, and selection problems [5], [6]. It provides mathematical tools for determining the number of possible outcomes under specified constraints.

One of the most fundamental concepts in combinatorics is combination, which measures the number of ways to select r objects from n distinct objects without considering order. The combination formula is given by (2) [5].

$$C(n,r) = n! / (r!(n-r)!) \quad (2)$$

where

n = total number of candidate leave days,

r = number of leave days selected,

and $n \geq r \geq 0$.

In the context of annual leave planning, combinations can be used to enumerate possible leave schedules. Suppose there are n candidate working days that could potentially be taken as leave and an employee is allowed to take r leave days. The number of possible leave schedules is therefore C(n,r).

D. *Optimization Concepts*

Optimization refers to the process of finding the best solution among a set of feasible alternatives. In annual leave planning, the objective is to maximize the total number of vacation days obtained from a limited leave quota.

Let V denote the total number of vacation days obtained after combining annual leave, weekends, and public holidays. V can be expressed as

$$V=L+W+H \quad (3)$$

where

L = leave days used

W = weekend days connected to the leave period

H = public holidays connected to the leave period

The optimization objective is to maximize V.

$$\text{Maximize } V \quad (4)$$

The feasible solutions are generated using combinatorial methods, while calendar information obtained through Zeller's Congruence is used to evaluate each solution. This formulation transforms annual leave planning into a combinatorial optimization problem [7].

III. METHODOLOGY

A. *Research Framework*

This research applies Zeller's Congruence and combinatorial analysis to optimize annual leave allocation. The methodology consists of four main stages:

1. Collecting calendar and public holiday data.
2. Determining weekdays using Zeller's Congruence.
3. Identifying candidate leave days located between weekends and public holidays.
4. Generating and evaluating leave combinations to determine the optimal leave schedule.

The overall objective is to maximize the total number of vacation days while satisfying the annual leave quota.

B. *Data Collection*

The data used in this study consist of:

1. Gregorian calendar dates for the selected year.
2. Indonesian public holidays.
3. Weekend information (Saturday and Sunday).
4. Annual leave quota available to an employee.

Public holidays and weekends are treated as non-working days, while weekdays that are not public holidays are considered working days. The Bank Indonesia holiday calendar was selected because it provides a consolidated list of public holidays and collective leave days for 2026 based on the applicable government regulations [8].

C. *Application of Zeller's Congruence*

For each date in the selected calendar year, Zeller's Congruence is applied to determine the corresponding weekday.

The resulting weekday information is then used to classify each date as either:

- Working day
- Weekend
- Public holiday

This classification enables the identification of strategic leave opportunities, particularly working days located between weekends and public holidays.

D. Combinatorial Model

After classifying all dates, candidate leave days are identified. Candidate leave days are working days that can connect multiple non-working days into a longer vacation period. For example, if a public holiday falls on Thursday, taking leave on Friday creates a four-day vacation period consisting of Thursday, Friday, Saturday, and Sunday. Such bridging days are considered candidate leave days for optimization.

E. Optimization Algorithm

Suppose there are n candidate leave days identified from the calendar analysis. If an employee has a leave quota of r days, the number of possible leave schedules is given by (2).

$$C(n,r) = n! / (r!(n-r)!) \tag{2}$$

Each combination represents a potential leave schedule.

Let

$$S = \{d_1, d_2, ..., d_r\}$$

denote a selected set of leave days.

For every possible combination S, the corresponding vacation period is calculated. The leave schedule that produces the maximum number of vacation days is selected as the optimal solution.

F. Leave Schedule Optimization

The optimization procedure is performed as follows:

1. Determine all candidate leave days.
2. Generate all feasible combinations using the combination formula.
3. Calculate the total vacation days obtained for each combination.
4. Compare all results.
5. Select the combination that produces the largest vacation duration.

The optimization objective is

$$\text{Maximize } V$$

subject to

$$|S| \le Q$$

where

V = total consecutive vacation days obtained,

S = set of selected leave days,

Q = annual leave quota.

G. Evaluation Metrics

The effectiveness of a leave schedule is measured using two metrics:

1. Total Vacation Days

The total number of consecutive vacation days obtained after combining annual leave, weekends, and public holidays.

2. Leave Efficiency Ratio

Leave efficiency is defined as

$$E = V / L \tag{5}$$

where

E = leave efficiency ratio,

V = total vacation days obtained,

L = number of leave days used.

A larger value of E indicates a more efficient leave schedule.

IV. Results and Discussion

A. Verification of Zeller's Congruence

To verify the correctness of Zeller's Congruence, the date of Nyepi Day, 19 March 2026, is used as a sample calculation. For the Gregorian calendar, Zeller's Congruence is given by equation (1).

$$h = (q + \lfloor 13(m + 1)/5 \rfloor + K + \lfloor K/4 \rfloor + \lfloor J/4 \rfloor + 5J) \bmod 7$$

For 19 March 2026,

q = 19,

m = 3,

K = 26,

J = 20.

Substituting these values into the formula:

$$h = (19 + \lfloor 13(3 + 1)/5 \rfloor + 26 + \lfloor 26/4 \rfloor + \lfloor 20/4 \rfloor + 5(20)) \bmod 7$$

$$h = (19 + \lfloor 52/5 \rfloor + 26 + 6 + 5 + 100) \bmod 7$$

$$h = (19 + 10 + 26 + 6 + 5 + 100) \bmod 7$$

$$h = 166 \bmod 7$$

$$h = 5$$

According to Zeller's Congruence,

0 = Saturday

1 = Sunday

2 = Monday

3 = Tuesday

4 = Wednesday

5 = Thursday

6 = Friday

Since h = 5, the calculated weekday is Thursday. This result matches the actual weekday of 19 March 2026.

TABLE I. VERIFICATION OF ZELLER'S CONGRUENCE RESULTS

Date	***Calculated Weekday***	***Actual Weekday***	***Match***
16 Feb 2026	Monday	Monday	Yes
19 Mar 2026	Thursday	Thursday	Yes
28 May 2026	Thursday	Thursday	Yes
24 Dec 2026	Thursday	Thursday	Yes

As shown in Table I, all tested dates produced weekdays consistent with the official Indonesian calendar. This confirms that Zeller's Congruence can correctly determine weekdays and can therefore be used as the basis for leave schedule optimization.

B. Identification of Strategic Leave Opportunities

After applying Zeller's Congruence to all dates in the selected year, several strategic leave opportunities were identified. One common pattern occurs when a public holiday is adjacent to a weekend. By taking leave on one or two working days located between these non-working days, employees can obtain significantly longer vacation periods. These bridging days become candidate leave days in the optimization process.

C. Combinatorial Analysis of Leave Allocation

Based on the 2026 Indonesian public holiday and Bank Indonesia’s collective leave calendar, several holiday clusters can be identified. The most notable clusters occur around Chinese New Year, Nyepi–Eid al-Fitr, Ascension Day, Eid al-Adha, and Christmas.

Assume an employee has an annual leave quota of five days. Candidate leave days are identified as working days adjacent to weekends, public holidays, and collective leave periods. Suppose that ten candidate leave days are obtained from the holiday clusters identified throughout the year. The number of possible leave schedules can be determined using the combination formula in (2):

$$C(10,5) = 10! / (5!(10-5)!) = 252.$$

There are 252 possible ways to select five leave days from ten candidate leave days. Each combination represents a potential leave allocation strategy and is evaluated based on the total vacation duration it produces. One of the combinations that yields the longest vacation period is

S = {17 March, 25 March, 26 March, 27 March, 30 March}

These leave days are strategically positioned around the Nyepi and Eid al-Fitr holiday cluster, creating an extended continuous vacation period. By evaluating all feasible combinations, the proposed method identifies the leave schedule that maximizes vacation duration while satisfying the leave quota constraint.

D. Optimal Leave Scenarios

One of the most advantageous holiday clusters in 2026 occurs around Nyepi and Eid al-Fitr. Based on the 2026 Bank Indonesia holiday and collective leave calendar, the official holiday schedule is as follows.

TABLE II. OPTIMAL LEAVE SCHEDULE AROUND NYEPI AND EID AL-FITR 2026

Date	***Status***
17 Mar 2026	Annual Leave
18 Mar 2026	Collective Leave (Nyepi)
19 Mar 2026	Nyepi
20 Mar 2026	Collective Leave (Eid al-Fitr)
21–22 Mar 2026	Weekend
23–24 Mar 2026	Collective Leave (Eid al-Fitr)
25 Mar 2026	Annual Leave
26 Mar 2026	Annual Leave
27 Mar 2026	Annual Leave
28–29 Mar 2026	Weekend
30 Mar 2026	Annual Leave

As shown in Table II, the holiday period can be extended from 17 March to 30 March 2026 by using five annual leave days. The resulting vacation duration is 14 days while only 5 leave days are used. The leave efficiency ratio is therefore

$$E = \frac{V}{L} = \frac{14}{5} = 2.8$$

Meaning that every leave day generates approximately 2.8 vacation days.

This result illustrates that strategic leave allocation around existing public holidays and collective leave periods can significantly increase vacation duration without requiring a large leave quota.

E. Comparison with Non-Optimized Leave Planning

To evaluate the effectiveness of the proposed approach, the optimized leave schedule is compared with a non-optimized leave allocation strategy. In a non-optimized scenario, employees may select leave days without considering the placement of weekends and public holidays. As a result, leave days are often scattered throughout the year, producing shorter and fragmented vacation periods.

For example, using five randomly selected leave days may generate only five to seven total vacation days, depending on their proximity to weekends and public holidays. In contrast, the optimized schedule identified in this study strategically places leave days around the Nyepi and Eid al-Fitr holiday cluster. By using the same five leave days, employees can obtain a continuous fourteen-day vacation period.

The comparison demonstrates that the proposed method significantly improves leave utilization efficiency. Therefore, mathematical approaches based on Zeller's Congruence and combinatorial analysis can provide practical benefits for annual leave planning.

F. Limitations of the Proposed Method

Although the proposed method successfully identifies leave schedules that maximize vacation duration, several limitations should be considered. First, the study assumes that employees are able to take leave on any candidate working day. In practice, leave requests may be subject to company policies, operational requirements, and managerial approval, which can restrict the availability of certain leave dates [1], [2].

Second, the optimization model focuses solely on maximizing the total number of consecutive vacation days. Other factors that may influence leave decisions, such as travel costs, personal preferences, family commitments, and workload considerations, are not included in the current optimization objective. As a result, the mathematically optimal solution may not always be the most practical choice for every employee.

Third, the results presented in this study are based on the 2026 Bank Indonesia holiday and collective leave calendar [8]. Any changes to government regulations, public holiday schedules, or collective leave arrangements may alter the candidate leave days and consequently affect the optimal leave allocation identified by the model.

G. Discussion

The results demonstrate that Zeller's Congruence can accurately classify dates into weekdays and weekends, providing the necessary calendar structure for leave optimization. The identified holiday clusters can then be analyzed using combinatorial methods to determine the most beneficial leave allocation strategy.

Among all holiday clusters in 2026, the Nyepi–Eid al-Fitr period offers the greatest opportunity for vacation extension because multiple public holidays and collective leave days occur within a short interval. Consequently, only a small number of additional leave days are required to obtain a long continuous vacation period.

The optimal leave schedule identified in this study enables an employee to obtain 14 consecutive vacation days by using only 5 annual leave days, resulting in a leave efficiency ratio of 2.8. This result indicates that strategic leave allocation can significantly increase vacation duration compared with non-optimized leave planning.

The proposed approach transforms annual leave planning into a discrete optimization problem. Rather than selecting leave dates intuitively, employees can use mathematical methods to systematically evaluate alternative schedules and maximize vacation duration under a limited leave quota. Although the study focuses on the Indonesian holiday calendar for 2026, the same methodology can be applied to other years and regions by adjusting the holiday data accordingly.

V. Conclusion

This paper presented an approach for annual leave optimization using Zeller's Congruence and combinatorial analysis. Zeller's Congruence was applied to determine weekdays and classify calendar dates, while combinatorial methods were used to generate and evaluate possible leave schedules. The optimization model identified leave allocations that maximize vacation periods under a limited leave quota.

The results demonstrate that mathematical techniques can support systematic and efficient annual leave planning. Future work may explore advanced optimization algorithms and automated calendar-based recommendation systems to improve scalability and practical implementation.

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PERNYATAAN

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